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## **SCIENTIFIC ORIENTATION OF MATHEMATICAL INSTRUCTION HISTORY AND CHANCE OF A GUIDING PRINCIPLE IN EAST AND WEST GERMANY**

### **1. The choice of the subject of the present address**

The opening address to the conference on mathematical education in Leipzig is meant to consider - in accordance with the wishes expressed by the organizing body - the evolution of mathematical education in East and West Germany throughout the last 30 years.

Among the large number of components that influenced the historical evolution of mathematical education, scientific orientation was a very powerful one.

We are going to deal only with this subject in the present address.

### **2. Scientific orientation and the great curriculum reform of mathematical instruction**

During the sixties and seventies a true revolution in the area of mathematical education evolved all civilized countries around the world. One joint idea of the great curriculum reform was shared by all such countries, namely to provide students with an appropriate picture of the cultural phenomenon of mathematics. The great curriculum reform of mathematical education was directed towards a scientific approach.

### **3. Political reasons for the scientific orientation and its consequences for the organization of school and teachers' training systems**

In both political systems in East and West Germany prevailing political reasons intensified the demand for scientific orientation of the entire educational system, inclusive of mathematics.

In West Germany, three axioms turned out to be decisive:

- (1) The well-being of the population is tightly connected to a high level of scientific education [ 28 ];
- (2) scientific education of the population is required to ensure proper functioning of a parliamentary democracy;
- (3) scientific education available for everybody is a major proof of the equal value of every citizen and thus helps create social justice.

As a rule, no educational subject should be withheld from part of the population.

As a consequence of this organisation, the elementary common school (Volksschule) was abolished in 1964. This type of school was- as indicated by its name - the school for popular education and vital practical alignment. This school was separated from scientific and intellectual education, which was the objective of advanced schools and universities. Popular education was also emotionally connected to the matters of the world. Providing daily practice and experience with a conceptual order and theory, which was the characteristic feature of intellectual education, was not deemed necessary. [ 3 ]

The elementary common school (Volksschule) was no longer up-to-date. It was replaced by the primary school (Grundschule) and carrying on from it, by the main school (Hauptschule), high school (Realschule) and grammar school (Gymnasium). A minimum school attendance during 9 years was compulsory.

In all these schools scientifically directed teaching was to be performed. The structure of teaching procedures was therefore modified. Instead of the teacher being in charge of a specific grade in the elementary common schools (Volksschule) who was to teach all subjects, the subject teacher was introduced into the main school (Hauptschule) too. The employ of specialized teachers was already customary in the high school (Realschule) and in the grammar school (Gymnasium).

Besides the main subjects, that is German and Mathematics, English language classes, instruction on labor issues (Arbeitslehre) and social studies were introduced for all types of pupils, even in the main school (Hauptschule).

Endeavours to start off with specialized teachers in the first grade of primary schools (Grundschule) failed.

A large minority of the western population felt that social integration of pupils educated in three different types of schools was inadequately achieved. Comprehensive schools (Gesamtschulen) were therefore set up, where residual parts of the three-fold schooling system remained visible in the standard classes (levels; Niveaueurse) where German, Mathematics and English were taught as major subjects.

The more exacting teaching objectives required a new type of training for the teachers. The teacher of the elementary common school (Volksschule) had been trained in training academies. These were awarded with the status of institutions of advanced teaching and were called "pedagogical colleges" (Pädagogische Hochschulen). They took on the task of training teachers for primary schools (Grundschulen), main schools (Hauptschulen) and high schools (Realschulen) as well as research activities in the areas of teaching-learning research on subjects taught in the aforementioned schools.

Therefore especially mathematical education was established in terms of a scientific discipline with independent professorships for mathematics and mathematical education.

In East Germany (GDR) only one school was set up, *the polytechnic high school* (Polytechnische Oberschule) with ten years of schooling for every student, where mathematical teaching comprised identical subjects for all students. The training of teachers expected to educate grades five to ten was ensured in universities and teachers' training colleges. Teaching-learning research in the educational area was carried out by professors for methodology of mathematical training/education.

#### **4. The consistent scientific orientation in East Germany**

In East Germany (GDR) scientific mathematical education was developed by the professors for methodology of mathematical training/education together with the academy of pedagogical sciences and the institute of school mathematics of the Humboldt University in East Berlin, further to debates that were controversial in many respects. The result was a consistent scientific orientation of mathematics, that is at present being taught at such universities.

Moreover the logical penetration of the entire mathematical teaching of the first to the tenth grade was provided.

The following examples may serve as proof for this statement [ 1, 2, 16, 26, 29, 32 ] :

1. As a rule, variables are used starting with the first grade.
2. The "If...then....form", which is often used in mathematics, is also generally used (example of the first grade: if  $x = 3$ , then  $x+5 = 8$ )
3. The introduction of multiplication of natural numbers is performed on the basis of the Cartesian product.
4. Geometric figures are understood as sets of points of the plane.
5. Proofs are introduced in the 6th grade in connection with theorems of the divisibility of numbers. They are cultivated also in a stern manner in the subsequent grades.
6. The positive rational numbers are introduced as equivalence classes of ordered pairs of natural numbers. The negative rational numbers are introduced as equivalence classes of ordered pairs of positive rational numbers.
7. The embedding of the numbers into the newly constructed comprehensive set of numbers is performed using an isomorphism.
8. The problems of the percentage-calculation and the "regel de tri" are solved with proportions and the formal notation of algebra.
9. Reflection, revolution and translation are understood as applications of the whole plane.
10. The trigonometry (computation of triangles) is introduced after definition of the functions  $\sin$  and  $\cos$  with the set of all real numbers as set of definition.
11. The terminology as employed at present at the universities is also consistently used in classrooms. The student is to be motivated with respect to the professional terminology. For instance: the plus-tasks are called addition tasks. The algebraic terminology such as sum, product, factor, dividend, divisor, quotient, term, commutativity of addition etc. are employed very early. German terms instead of words of Latin origin were prohibited.

This consistent scientific orientation, for which also some of the teachers at grammar schools (Gymnasium) in West Germany demonstrated positive feelings, was partly retracted in the eighties. Particularly the introduction of multiplication with the Cartesian product of sets as well as the introduction of the positive and negative rational numbers as equivalence classes were slightly retracted. The integration by means of isomorphisms vanished. Percentage tasks and "regel de tri" tasks were also solved by operations using the passage into the factual situation. [ 5, 27, 29 ]

The consistent scientific orientation, however, remained present as an important idea.

This idea is also effective in the eastern part of the reunified Germany. Many teachers of mathematics do not use certain terms just because they do not form part of the terminology of university mathematics.

## **5. The scientific Orientation of Mathematics at the Grammar School (Gymnasium) in West Germany**

The scientific orientation of mathematics at grammar schools (Gymnasium) was unobjected. However, a unique debate evolved about its type . Besides the *consistent scientific orientation* a different kind was under discussion, called the *genetic scientific orientation*, which

comprises also an orientation towards the intellectual power of the students and towards a naturally evolving learning and perception process. [ 11 ]

At the base level there are imaginations instead of concepts fixed by conditions. These imaginations are the basis for thinking at that level. [ 15 ]

Instead of formal proofs, there is intuitive insight into obvious situations or insight bound to facts (preformal proofs). [ 15, 25 ]

Important is an orientation towards the proper substance of mathematics. That is genetic scientific orientation and not the tendency to building up a terminology in mathematics that is at present employed at universities. [ 11 ]

## 6. Pedagogically orientated subject-analysis

If teaching of mathematics is consistently directed towards the scientific approach, then the positive rational numbers must be introduced as equivalence classes of natural numbers. If the instruction is genetically directed towards the scientific approach, then the teacher may divide a pizza or a cake into for instance 6 parts and call each part one sixth of the whole pizza or cake. May we call this type of teaching scientific? Is this mathematics at all ? One can enter into the same debate with regard to the customary instruction of the following subjects: introduction of quantities and calculation using them, introduction of negative numbers and calculation using them, "regel de tri", percentage-calculation. The question whether the customary instruction of these subjects was scientifically justified was one of the major challenges within mathematical education in West Germany in the sixties and seventies. [ 6, 7, 8, 9, 10, 11, 18, 20, 21, 22, 23, 24 ]

The analysis of mathematical subjects was no concern insofar as popular education was concerned. The first mathematician to modify this point of view was Walter Breidenbach who taught at the teachers' training college at Osnabrück. Background theories emerged that were supposed to demonstrate that the subject had a mathematically unobjectionable background and that teaching of this subject could be reasonably called scientific. Different background theories were built up for fractional numbers, in accordance with which the fractional numbers were introduced as operators [ 6, 7 ] or as numbers of measurement [ 8, 10, 18 ] or quasicardinal as numbers of fractional parts [ 9 ].

The decision as to which of these background theories were appropriate for a curriculum cannot be made by means of mathematical analysis, it requires the application of pedagogical, psychological and educational aspects. Therefore such background theories were also called *didactically (pedagogically) directed subject\_analysis*, because they depend on didactic decisions [ 11 ].

The didactically (pedagogically) directed subject analysis also serves the purpose to assist with precise finding of objectives and to solve problems inherent to subject education.

There were also didactically directed background theories on Algebra [ 12 ]. The purpose was to make instruction of Algebra more sensible and more effective. Also the use of helps, for instance the use of the weighing machines to substantiate the rules for transformation of equations, should be shown as scientifically admissible.

## 7. Scientific orientation of teaching of mathematics in primary schools

The former elementary common schools (Volksschule) governed by the idea of popular education ignored the subject of mathematics. The corresponding subject was called "arithmetics" (Rechnen) and the summum of the arithmetic subject was calculating in factual situations, thus applying the arithmetic operations to facts and figures of every-day-life.

With the foundation of primary schools in 1964, the idea of scientific method was not automatically applied to this type of school. A debate on this issue was going on. Speaker in favor of a scientific orientation was Arnold Fricke (teachers' training college at Braunschweig), but also Heinrich Winter (teachers' training college at Neuß and later Dortmund).

Arnold Fricke was the first to call his textbook in 1967 "Mathematics in primary school (Mathematik in der Grundschule) [ 4 ]. This book was written in cooperation with Heinrich Besuden (teachers' training college at Oldenburg). Arnold Fricke specified that teaching of arithmetics should also be performed in the spirit of mathematics. He insisted that the net of relations between numbers, arithmetical operations and equations was typical of mathematics. Therefore in his textbook the net between the arithmetical issues was emphasized and integrated by solving the arithmetical tasks.

Fricke and Besuden also derived their opinions from the psychology as interpreted by Piaget. To Arnold Fricke, problem solving was also essential in mathematics. He requested therefore that in primary schools also mathematical thinking had to start off from motivating problematic situations that had to produce solving attempts.

The problematic situation itself must not emerge from a factual situation but may be often more convincing from an intrinsic mathematical question. This aspect of an intrinsic mathematical problematic situation was entirely novel. The teaching in elementary common schools (Volksschulen), especially when directed at reform-pedagogical principles, would not work out. Only the instruction in grammar schools (Gymnasien) should stem from such intrinsic mathematical problems.

An extremely consistent scientific orientation was advocated by Heinz Schlechtweg (teachers' training college at Duisburg). He requested that those structures that he felt to be essential for mathematics be taught in primary schools, too [ 30 ].

This opinion was totally excessive. He proposed to introduce multiplication of natural numbers on the basis of Cartesian product of sets. This could hardly be achieved by any teacher.

An opponent to scientific orientation of arithmetic was Karaschewski (teachers' training college at Bielefeld) [17 ]. Karaschewski feels that mathematics consists of formal proofs. Therefore mathematics cannot be taught in primary schools. Karaschewski's idea of mathematics is very narrow. He focusses on one component only out of a spectrum of components that distinguish mathematics. According to this narrow idea of mathematics, geometry as taught in grammar schools (Gymnasium) (for instance geometrical construction of triangles or bodies) were no mathematics, as they are not substantiated by formal proofs.

## **8. Science-orientated teaching in main schools (Hauptschulen)**

The main school (Hauptschule) comprises grades 5 to 9. In grades 5 to 8 of the former elementary common school (Volksschule), the following calculating subject matters were dealt with: fractions, percentages and calculation of interest, account of settlement, ratio calculation, calculation of mixing, calculation of area or volume in two- or three-dimensional geometry and calculation in factual situations. Intra-mathematical problem configurations hardly existed. The theory of divisibility was integrated into fraction calculation and taught insofar only as this was required for simplifying of fractions and determination of common denominators.

Given the fact that in principle no mathematical contents currently taught at grammar schools (Gymnasien) were to be withheld from pupils, the following subjects were integrated in addition to the range of subjects as taught at main schools (Hauptschulen): geometrical transformations (reflections, rotations, translations), the theory of geometrical figures (triangles and quadrangles) independent from calculation in two- or three-dimensional geometry, the angle theorems up to the theorem of Thales, the theory of similitude and the centric stretchers. However, proofs were neglected and intuitive insight was cultivated instead. The theory of divisibility was taught, without taking into account fractions in terms of an independent mathematical section.

A new section of arithmetics was introduced, i.e. negative numbers and calculation with rational numbers as well as elements of probability and statistics. Algebra (from transformation of terms through to binomial formulas, equations through to linear equations with two variables and the quadratic equations and linear functions with their graphs) were integrated to a large extent into the range of subjects.

Working with algebra comprises necessarily working in the purely signitive sector which caused, as experience had taught, problems comprehension to pupils of main schools (Hauptschulen), differentiated algebra curriculums of varying levels were now developed. The lowest level comprised only dealing with calculation formulas with positive numbers or quantities as customary in professions and professional schools (Berufsschulen). In principle, the pupils of main schools (Hauptschulen) were to be faced with reductions of the subject in case they were likely to be unduly challenged otherwise. The minimum objective of mathematical teaching in main schools (Hauptschulen) was to be the creation of a basis for dealing with every-day-life-situations in terms of calculating as well as with relevant requirements for professional purposes and professional schools (Berufsschulen). Developing mathematical curricula for main schools (Hauptschulen) and solving of differentiation problems arising therefrom were one among the major challenges in western mathematical teaching in the seventies. Left aside authors of textbooks, only very few mathematical didacticians had a close look at this problem. A minute number of publications of western mathematical didacticians for main schools (Hauptschulen) is available - which is not a glorious issue in terms of western mathematical didacticism. However, the impact of the problem is reduced as main schools (Hauptschulen) find ever less supporters.

## **9. High schools (Realschulen)**

The high school (Realschule) in terms of close-to-practice technical and professionally orientated school has always been guided along with the curriculum of grammar schools (Gymnasium) and introduced only a few deviations from it. This tendency goes increasing

during the sixties and seventies. At present symptoms are to be distinguished that would rather suggest an inversion of such tendencies.

### 10. Why no genetic orientation of science in the East?

The response to the question in the heading may be explained by the refusal of reform teaching science in the East. The *guiding role* of the teacher was assumed to be rather more supporting and effective. When faced with the alternative of either guiding or leaving room for children's spontaneity and individual dynamics, it was guidance by the teacher that was adopted, maybe also because supervision in the other event would have been more difficult to achieve.

A further reason is that eastern psychology underestimated the signification of individual activity in the learning process.

### 11. Large or narrow notion of mathematics in the East?

Was a narrow or wide notion of mathematics customary in the East? As far as I know, no explicit statements are to be found in the eastern magazin „Mathematics in School“. If we recall response by the teachership to the challenge from the West further to the reunification process, we might assume that they had a narrow notion of mathematics.

However, there are also examples that may be quoted as proofs of the opposite. They stem from Klaus Härtig (Professor for Logics at the Humboldt University Berlin and leading as regards the reform of mathematical teaching in the East in the sixties and seventies). The first example deals with the proof of the equation for binomial coefficients. Härtig argues in connection with factual matters: "In a forest with  $n$  trees,  $k$  trees are to be cut down. A forester

$$\binom{n}{k} = \binom{n}{n-k}$$

has two alternatives how to indicate this to the lumberjacks. Either he marks the trees to be cut down or he marks the trees that are not to be cut down. Each marking of one type goes with exact one marking of the other type. The number of possibilities to cut down  $k$  trees out of  $n$  trees is equal to the number of possibilities not to cut down  $n - k$  trees."

The equation is therefore:

$$\binom{n}{k} = \binom{n}{n-k}$$

This is a traditional example for reasoning in terms of factual situations. It is also legitimate mathematics.

The second example by Härtig deals with the notion of mathematical strictness. At a lower level, strictness only means correctness. Härtig quotes as example the entirely correct explanation of the notion of a prime number in a first grade [14.] A prime number is a number that does not emerge from any "real" multiplication task. Multiplication tasks with the factor 1 are no "real" problems. The mathematician would say "trivial" instead of "real".

Mathematics do not only comprise the formal formulation.

Further examples quoted by Härtig could be indicated [ 14 ]. They show that the tendency for a wide notion of mathematics in the East existed - at any event - in the eighties.

## 12. Outlook

May scientifically orientated teaching look forward to a future, after all? Should not more widely opened teaching methods be advocated instead, where the pupil is the center of teaching efforts?

Should teaching not be rather seen from the pupil's view and take into account children's individual initiative and fun in discovering and their participation in responsibility of their own learning process be reinforced? In the West such ideas emerge ever more frequently.

Doubtless teaching orientated mainly towards subjects and their scientific contents faces the risk of disregarding learning interests and suggestions of the children and not to take sufficiently into account the individual prerequisites for learning, previous experiences and capabilities of the pupils.

On the other hand, in some elementary school magazines, examples of teaching multiply where nothing can be traced as regards the substance of mathematics.

A method of teaching orientated mainly towards the pupil is running a risk of turning into pure actionism, where the learning progress of pupils is ignored and of becoming dependent on situative coincidences [ 33 ].

As children modified their attitude during the last 25 years, teaching is becoming increasingly difficult. The teacher is faced with more immediate problems than scientific orientation and grasps the straw of pure activism as a way out.

One of the major problems of mathematical teaching of the future is therefore to develop curricula apt to make learning the individual problem of the pupil, with free room for participation in responsibility and organization of lessons by the pupil.

It would be a catastrophe for our mathematic discipline if mathematical teaching were no longer orientated towards science. Please help, colleagues at schools and universities to prevent this and to make sure serious mathematical knowledge and skill remain the trademark of our discipline.

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